Technical Comments

Errata: "Computer Analysis of Asymmetrical Deformation of Orthotropic Shells of Revolution"

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In the above article replace 1) $C_1^{(\ )}$ by $C_1^{(2)}$ in the third equation of Eqs. (1); 2) R by R_2 in the first equation of Eqs. (5); 3) $Z_{1_0}^{-1}$ by $Z_{1_1}^{-1}$ in the first and second equations of Eqs. (17); and 4) Ω_i by $\boldsymbol{\Omega}_i$ in the fourth equation of Eqs. (17).

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Comment on Non-Newtonian Flow

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Ablation materials of greatest practical interest, organic and inorganic glasses, have a strong tendency to non-Newtonian flow behavior. The occurrence of gas bubbles, caused by boiling, degassing, or decomposition of the substrate promotes also non-Newtonian flow properties. Non-Newtonian behavior is related to flow-induced structural changes within the liquid, caused by the orientation or degree of entanglement of very large molecules. In Ref. 1 the Ostwald-de Waele power-law was chosen:

$$\tau = a(du/dy)^n \tag{1}$$

to describe the non-Newtonian properties of the fluid. This law is valid for isothermal conditions, but it is a fact that the zero shear viscosity of practically all substances with non-Newtonian flow behavior is a strong function of the temperature. In all cases where temperature gradients occur throughout the liquid layer, as in ablation, the Ostwald-de Waele power-law will give wrong results. In order to treat the problem of non-Newtonian, nonisothermal flow of liquids, the laws of the temperature and shear rate dependency of liquids must be combined, and the boundary-layer equations must be integrated with this expression for the viscosity. The generally accepted relationship for the temperature dependency of the visocsity is

$$\mu = \exp(A/T - B) \tag{2}$$

This dependency is also true for non-Newtonian liquids as long as the zero shear viscosity is under consideration. The zero shear viscosity will, in the following, be expressed by μ. Eyring[2] and Bueche[3] developed expressions for the shear dependency of non-Newtonian liquids. For not too large deviations from Newtonian flow, they arrive at the following equation:

$$1/\mu_\tau = (1/\mu)(1 + k\tau^2) \tag{3}$$

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where μ_τ is the viscosity at the shear stress τ, and k is a constant, determining the degree of deviation from Newtonian flow properties. If k is negative, the fluid is rheopex, and with a positive k the fluid is thixotropic. By writing Eq. (2) in the form

$$\mu/\mu_i = (T/T_i)^{-n} \tag{4}$$

where μ_i is the zero shear viscosity at interface temperature and the power factor for n is defined by $n = A/T$, the expression for the shear and temperature dependencies of the viscosity can be combined to the following expression, which may be handled easily in the subsequent integration process:

$$\frac{1}{\mu_\tau} = \frac{1}{\mu_i}\left(\frac{T}{T_i}\right)^n (1 + k\tau^2) \tag{5}$$

By neglecting the inertia terms, which is always justified for non-Newtonian liquids, the liquid-layer equations near the stagnation point have the form

$$(\partial/\partial x)(ru) + (\partial/\partial y)(rv) = 0 \tag{6a}$$

$$\frac{\partial p}{\partial x} = \frac{\partial}{\partial y}\left\{\mu_i \left(\frac{T}{T_i}\right)^{-n} \frac{1}{1 + k\tau^2}\frac{\partial u}{\partial y}\right\} \tag{6b}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \tag{6c}$$

The only deviation from the well familiar liquid-layer equations appears in Eq. (6b) where the viscosity coefficient is replaced by a viscosity function $\mu(\tau, T)$. The integration of Eqs. (6a–6c) for the vicinity of the stagnation point, where not only the shear stress but also the pressure gradient contribute to non-Newtonian behavior, is straightforward and leads to the following result:

$$\mu_i v_w{}^4(v_w - v_i) = \frac{2a\alpha^2}{n^2}v_w{}^2 - \frac{4b\alpha^3}{n^3}v_w + x^2k\left\{\frac{4a^3\alpha^2}{n^2}v_w{}^2 - \frac{24a^2b\alpha^3}{n^3}v_w + \frac{72ab^2\alpha^4}{n^4}\right\} \tag{7}$$

v_w is the total ablation velocity, v_i the ablation velocity by evaporation, $a = \partial\tau/\partial x$, $b = \partial^2 p/\partial x^2$, and α is the thermal diffusivity of liquid. For $k = 0$ the result is equivalent to that obtained by Bethe and Adams.[4] Equation (7) shows that for $x = 0$, that is, at the stagnation point, there is no modification of the flow by non-Newtonian behavior, but deviations increase with x^2 in the vicinity of the stagnation point. The value for k changes over several orders of magnitude for various substances from zero to 10^{-2}. In order to treat the ablation behavior of non-Newtonian liquids, the heat-transfer conditions have to be known in addition to Eq. (7). In case the flow of liquids containing gas bubbles has to be treated, k does not, of course, express structural changes within the liquid, but an experimentally determined proforma value for k may be used.

More comprehensive calculations about the ablation of non-Newtonian liquids are now being carried out and will be reported.

References

[1] Wells, C. S., "Unsteady boundary-layer flow of a non-Newtonian fluid on a flat plate," AIAA J. 2, 951–952 (1964).

[2] Eyring, H., "Viscosity, plasticity, and diffusion as examples of absolute reaction rates," J. Chem. Phys. 4, 283 (1936).